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TABLE OF CONTENTS

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The elaboration of the Giza site-plan.

R. J. Cook

As far as the present writer is aware no arguments have yet been advanced which would show that he or John Legon, author of a series of articles in this journal on the subject of pyramid geometry, are wrong in concluding that the Giza group was laid out to an overall site plan. However any description of this plan must explain why this group was arranged in its particular configuration, yet Legon rejects the proposition advanced by Bauval (1) that the placing and relative proportions of the three pyramids were primarily determined by the desire to reproduce the configuration and magnitudes of the stars of Orion's belt. Instead he appears to hold fast to his earlier contention that the three pyramids were laid out using simple geometry and that the final configuration was intended to express the roots of 2, 3, and 5, without explaining why such an aim should have motivated a people which all the evidence shows were obsessed by religion and preparation for the afterlife.

Now while the belt-star correlation is remarkably good it is not exact and, since it can be shown that the final dimensions and positions of the Giza pyramids are indeed determined geometrically, it would equally be futile to discuss their meaning without paying full attention to the geometrical aspect. It should not be assumed however that geometry was employed for purely practical reasons - for example merely to provide the basis of a modular grid to facilitate the setting out of the design - and the purpose of the present paper is to highlight those facets of the geometry which, according to the writer's contention, would more likely have represented a symbolic counterpart to the religious beliefs motivating the architect. The figures illustrating the following description are orientated with south at the top in conformity with the ancient Egyptian viewpoint.

* * *

In a recent paper (2) the present writer described the salient points of a geometrical scheme uniting the design of the pyramid of KHUFU with an overall site plan for the three Giza pyramids. Beginning with a 'model pyramid' of proportions base 2 units and height 1.2727 units (hence with height-to-base ratio 7/11), exhibited in the design of the KHUFU starshafts at a scale of 1/100, and in the layout at the scale of 1/1000, it was shown how the dimensions of KHUFU (height 280 and base 440 cubits), and the overall dimensions of a modular plan described by John Legon (3), are derived from this essential scheme. In addition a remarkable correlation between this geometry and the altitudes of cult stars at the time of building was described.

Legon's plan of the Giza pyramids, based upon Petrie's survey data (4) and a royal cubit of 0.5237 m., is developed from the northeast corner of KHUFU to the southwest corner of MENKAURA and has overall dimensions 1732 cubits north/south and 1417.5 cubits east/west and Legon shows, by reference to the placing of KHAFRA and other features, how it is developed from a basic unit of 1000 cubits. But whereas the north/south dimension clearly equals $\sqrt{3}$ X 1000 cubits, Legon's insistence that the east/west dimension is intended as an approximation for $\sqrt{2}$ X 1000 (1414) cubits is somewhat at variance with the meticulous attention to detail that characterizes much of his work (5).

Figure 1 shows a square of side 2500 cubits within which a simple construction is made employing the two angles 26.5° (the semi-diagonal of a square) and 60° (the diagonal of

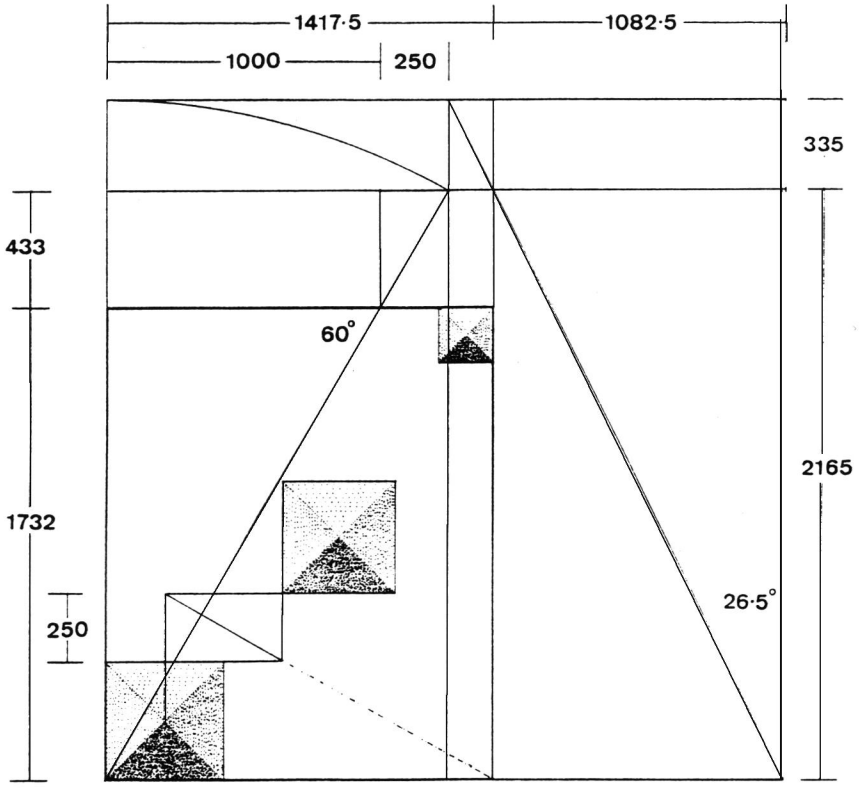

Fig. 1

a $\sqrt{3}$ rectangle) to give the quantities 1417.5 (the east/west dimension) and 2165 cubits, which is in turn multiplied by 4/5 to give the north/south dimension of the plan. The $\sqrt{3}$ rectangle measuring 250 X 433 cubits isolated by this operation is particularly interesting for we see that it is also defined between KHUFU and KHAFRA, the latter dimension being composed of the quantities 220 and 213 cubits.

In his description Legon begins by identifying an 'initial' modular scheme for the layout of the two large pyramids. The present writer's interpretation of it is shown in figure 2. The north/south dimension of this scheme is 1100 cubits (subsequently modified to 1101 cubits) while the east/west dimension is 1065 cubits (modified to 1064 cubits). It is clear that both dimensions are derived from a simple geometric operation within a square of side 1230 cubits involving once again the angles 26.5° and 60°, and in turn defining a rectangle ('A') of dimensions 130 X 165 cubits in the northwest corner of the figure. It is interesting to note that the proportions of this rectangle are very closely approximated in the rectangle ('B') formed between the centres of the two pyramids (6). Note also that this geometric scheme is reiterated at a scale of one fifth, and centred on the west side of KHUFU, to account for the east/west spacing between the two pyramids of 213 cubits.

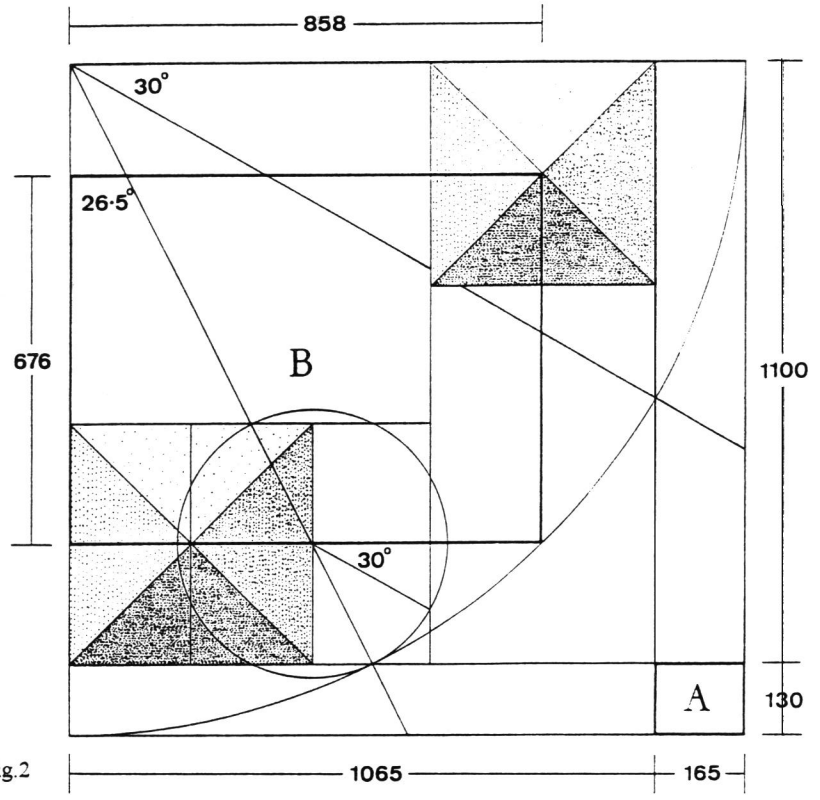

Fig.2

This scheme forms the basis for an initial scheme for KHUFU developed by the present writer (7) and shown in figure 3. The circle of radius 246 cubits is centred on the great step at a level 82 cubits above pyramid base (this level being determined by the well-known geometric construction on the height of the pyramid as shown to the right of the figure). The line at 26.5° represents the general line of the upper passages and, in the left upper quadrant, the reiteration of the previous figure produces the quantities 220 and 213 cubits and defines a rectangle ('C') measuring 26 X 33 cubits (one fifth the scale of rectangle 'A' of the previous figure) and displayed in the dimensions of the King's Chamber complex - the south wall of the King's Chamber lying 26 cubits from the east/west plane of the pyramid, while the uppermost (5th) granite ceiling of the King's Chamber lies 33 cubits above its floor. Note also in this scheme that if the line of the upper passages is extended to the south wall of the King's Chamber then the total passage length from this point to the intersection with the descending passage, and from here to the entrance, measures 246 cubits.

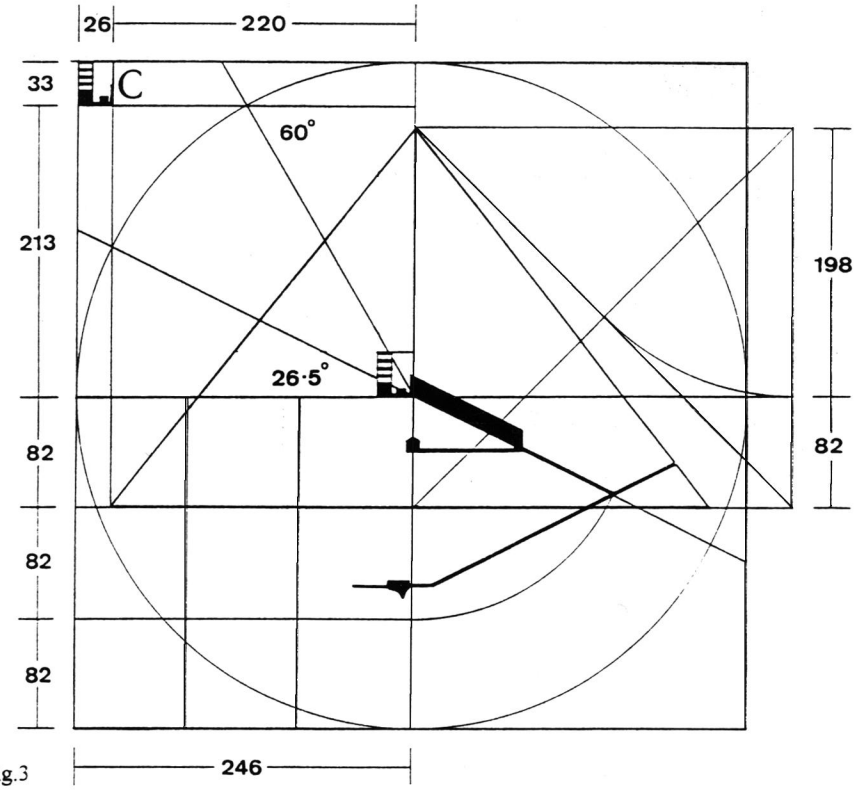

Fig.3

The initial layout plan, five times the scale of the initial KHUFU scheme, is further developed in figure 4, but now the $\sqrt{3}$ dimension 2165 cubits is constructed along the east/west axis of the square (the quantity 335 cubits being subtracted from the south and added to the north in the manner shown) and is seen to be divided by the west side of KHAFRA into the two parts 1065 and 1100 cubits. Consequently the north/south dimension 2500 cubits may be divided into the two parts 1230 and 1270 (2 X 635) cubits by the east/west axis passing through the centre of KHAFRA. The highlighted rectangle measures 2165 X 1250 cubits, divided in the same manner as the rectangle measuring 433 X 250 cubits discussed in figure 1, and its southern edge locates the north side of MENKAURA.

Before describing the geometrical arrangement which replaced this initial scheme, reference must be made to an elegant 'circle-squared' geometry for the dimensioning and placing of MENKAURA described by Legon (8), and shown inset in figure 5. A square of side 500 cubits is placed diagonally within a circle and then a square is drawn equal in perimeter to the circumference of the circle, hence with side equal to $125\pi\sqrt{2} = 555.36$ - this figure being rounded to 555 cubits. The intersections of the two squares produce the dimension 201.5

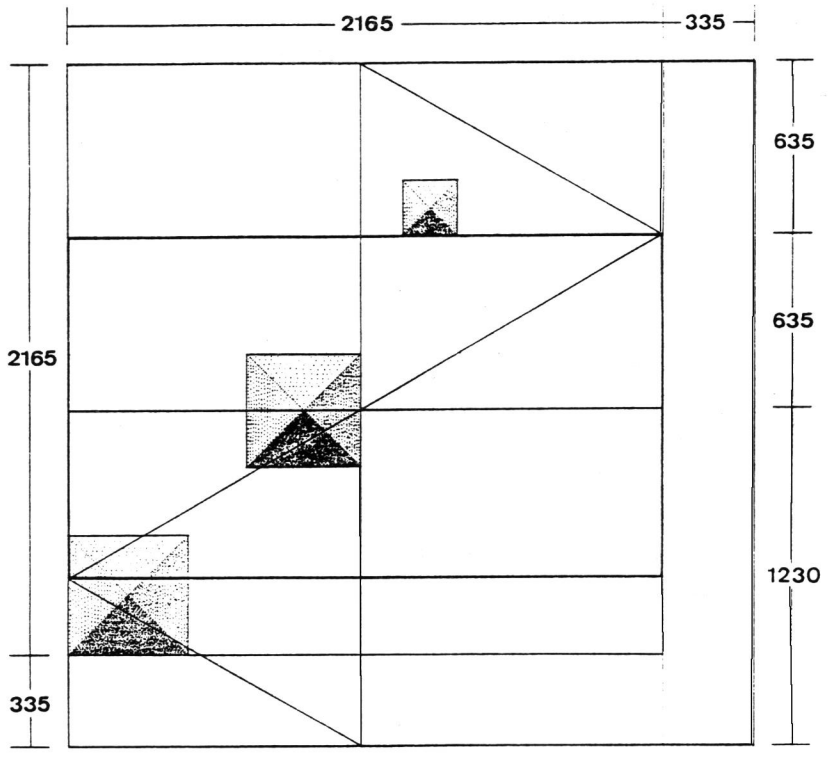

Fig.4

cubits for the base of MENKAURA. This figure is placed as shown such that the south side of this pyramid lies 631 cubits from the south side of KHAFRA (or 1101 cubits from the north side of KHUFU), while its east side lies 152 cubits from the west side of KHAFRA. The base of KHAFRA is effectively defined as $500(\sqrt{5} - \sqrt{2}) = 410.93$ cubits, rounded to 411 cubits, as opposed to the 410 cubits of the initial plan.

Now if this scheme is expanded by a factor of four, such that the circle has a radius of 1414 cubits and the side of the square 2220 cubits, and the layout plan drawn within the square in the manner shown, then the dimension 201.5 cubits becomes 806 and the dimension 152 becomes 608 - and the east side of KHUFU lies 2 X 608 cubits from the east side of MENKAURA. It is important to recall that these are rounded figures, but this does not make them any the less valid - we are simply dealing with whole-number proportions. For instance, we may express the ϕ ratio geometrically or it may be approximated by the ratio between two terms of any additive series in which each term is the sum of the preceding two. eg:

1, 4, 5, 9, 14, 23, 37, 60, 97, 157, 254, 411...

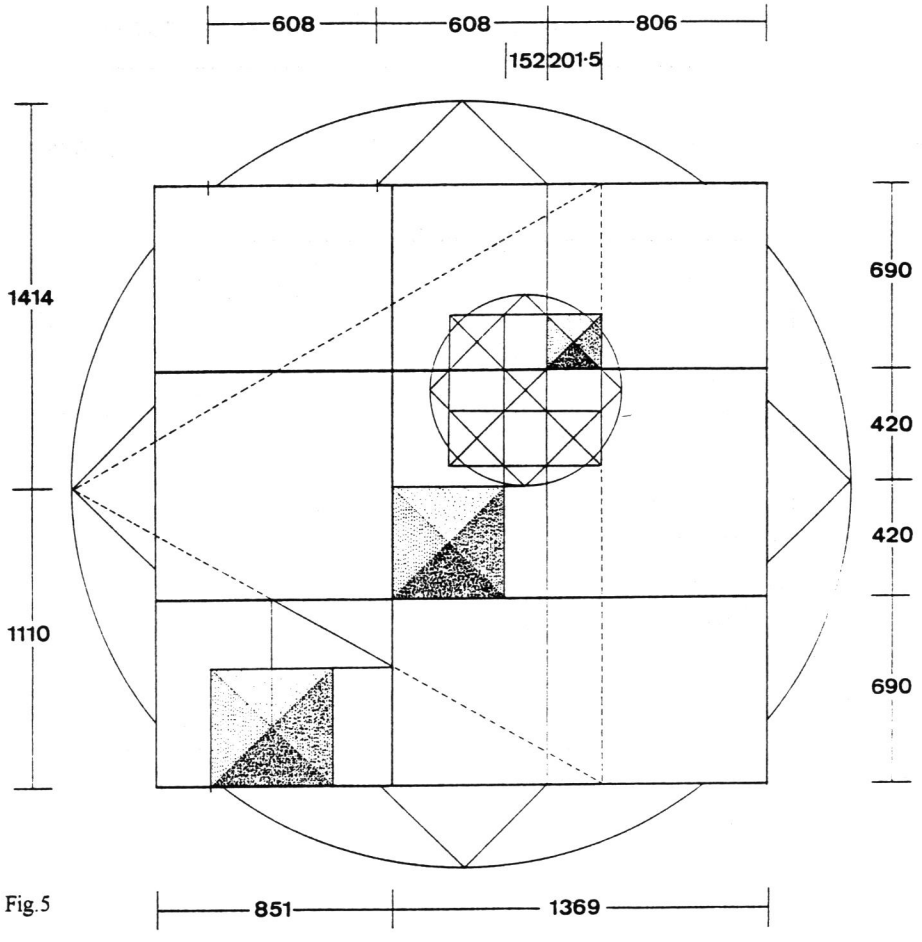

Fig.5

In our figure the dimension 2220 cubits equals 60 X 37. The east side of KHAFRA lies 1369 cubits from the west side of the square, representing a division in the ratio 23 : 37. Similarly the north/south spacing between the north sides of the three pyramids may conveniently be described by the ratio :

23 : 14 + 14 : 23. (690 : 420 + 420 : 690).(9).

The dotted lines indicate an equilateral triangle which may be drawn within this figure which, although not absolutely precise, would seem to bear some significance - its north side forms part of a general alignment through the $\sqrt{3}$ rectangle seperating KHUFU and KHAFRA.

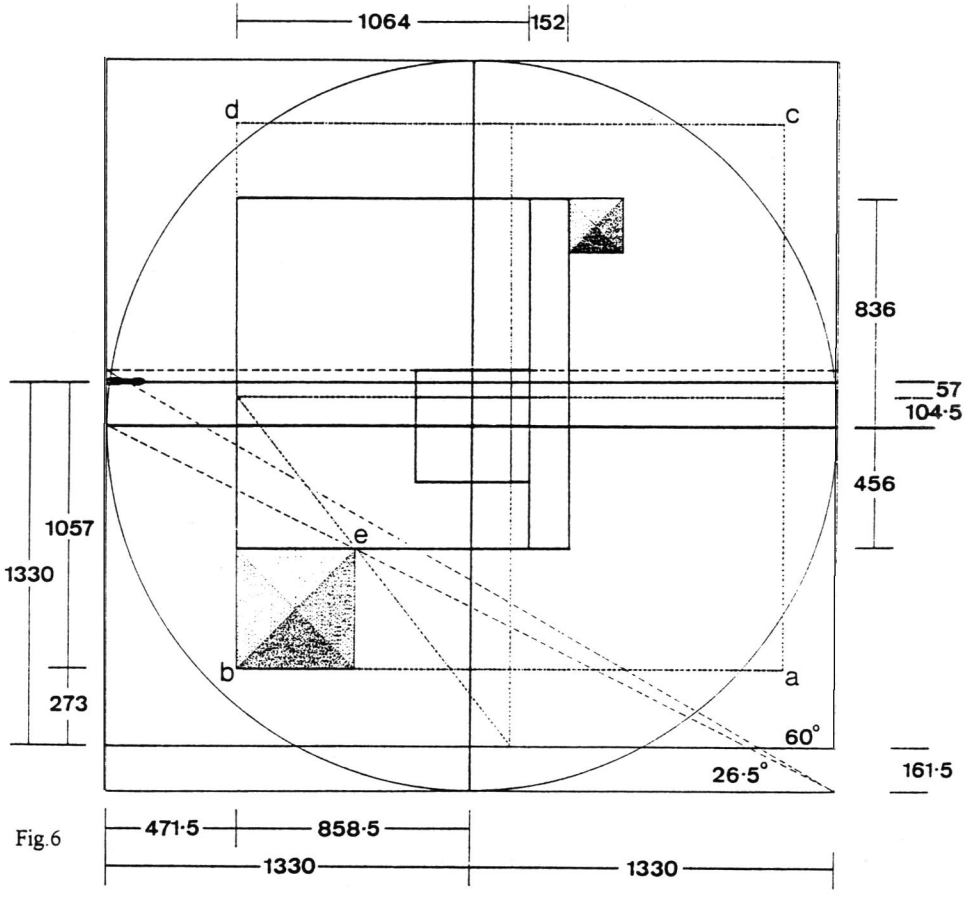

Fig.6

The geometrical arrangement which replaced the initial scheme is developed from the layout square ABCD of side 2000 cubits and reproduced as part of figure 6. As proposed in the writer's previous paper, this square represents the base of a 'model pyramid' of height 1272.72 cubits (or in proportion 7/11 with the base). When the meridian section of this pyramid is superimposed upon the base, then the 'side' of the pyramid intersects the diagonal of the square at point E to locate the southwest corner of KHUFU and define its dimensions.

Now it is well-known that the proportion 7/11 can be closely approximated as $2/\pi$ or $\sqrt{\phi}/2$ (so that the height of our model pyramid will measure 1273.24 and 1272.02 cubits respectively). The 1272 value produces the well-known ϕ proportion between base and apothegm for KHUFU : 272 + 440 = 712. If the height of our model pyramid is made 1273 cubits an interesting result is obtained.

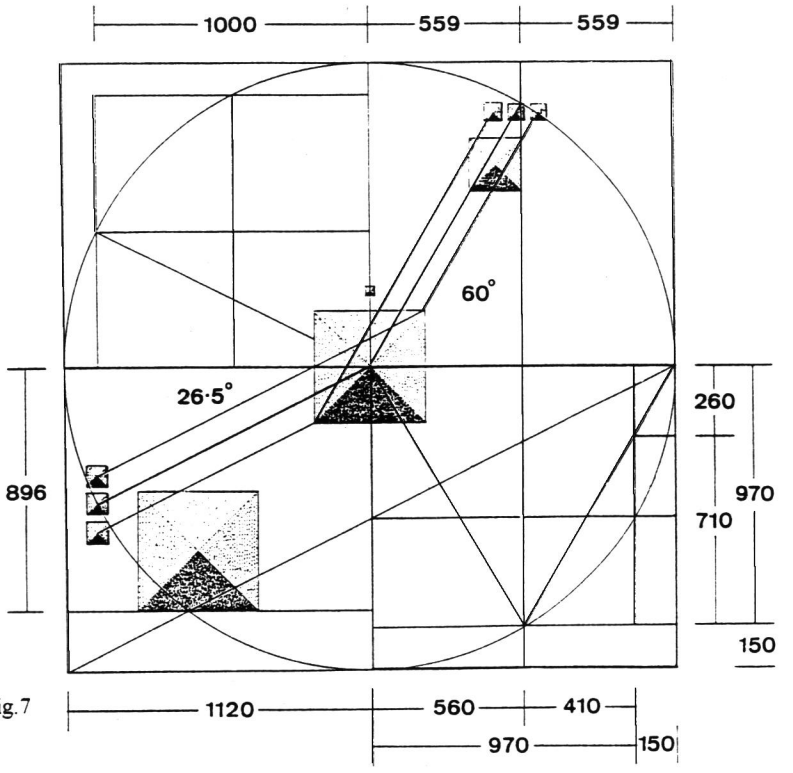

Fig.7

Legon locates the axis of the Sphinx 1057.14 cubits (or 7400 palms) south of the north side of the plan (10). If this dimension is taken as 1057 cubits then the distance from the Sphinx axis to the 'apex' of the layout pyramid becomes 1273 + 57 = 1330 cubits. The east side of the Sphinx he locates 471.43 cubits from the east side of the plan. If this dimension is taken as 471.5 cubits and added to the 858.5 cubits seperating the east side of the plan from the north/south axis of KHAFRA then we again obtain the quantity 1330 cubits. This dimension equals 5/4 X 1064 cubits - the quantity we have already met in the previous figure.

It is clear that we are dealing with a further circle squared scheme in which all quantities are multiples of 19. The north/south distance seperating the south sides of KHUFU and MENKAURA is 1292 cubits and according to this scheme is divided into the two parts 456 and 836 cubits by the east/west axis of KHAFRA. 456/2 = 228 and 228 + 836 = 1064. 1064/836 = 1.2727.

This scheme is effectively reiterated at smaller scale 'within' KHAFRA : the south side of the square of 1000 cubits lies 104.5 (836/8) cubits south of the east/west axis of KHAFRA while the Sphinx axis lies 161.5 (1292/8) cubits south of this same axis, or 44 cubits north of the south side of the pyramid (a dimension reflected in the semi-width of the mortuary temple of 44 cubits). Note that, as given by the construction described in the writer's previous paper, 1292 X $\sqrt{3}$ = 2238 cubits : the diameter of the circle enclosing Legon's plan.

Finally we have two alignments : the 26.5° semi-diagonal passing through point E, and a 60° alignment from the northwest corner of the figure to reach the east side of the figure at a point locating the south side of KHAFRA. This last closely parallels the diagonal of the $\sqrt{3}$ rectangle defined between KHUFU and KHAFRA.

This description by no means includes all of the geometrical relationships so far proposed for the Giza monuments, but I hope to have demonstrated that rather than one single plan we find a multiplicity of superimposed schemes. Neither can we insist that the replacement of an initial plan was dictated
by the desire to make the dimensions of the final plan more closely approximate root ratios. We do however find the repeated use of simple geometric operations involving such roots, and the angles 26.5° and 60°, in the establishment of these geometric schemes.

Figure 7 displays a geometric scheme centred on the pyramid of KHAFRA put forward by the present writer in 1985. The suggestion that the subsidiaries of KHUFU were built to the east of the main pyramid because the quarry and access ramp lay to the south (the usual position for such subsidiaries) appeared unconvincing. Upon investigation it was found that parallel alignments could be drawn, with sufficient accuracy to seem significant, from the centre and corners of KHAFRA to the centres of the KHUFU subsidiaries, and also to the three subsidiaries of MENKAURA - the former at an angle of 26.5° and the latter at 60°. It seemed as if these subsidiaries were placed to draw attention to an underlying geometry, or to act as symbolic alignments (as recently confirmed by Bauval's observation that the azimuth 26.5° north of east was the angle of the sunrise at the heliacal rising of Sirius (rising point 26.5° south of east) at epoch 2450 BC., and thus had cultic significance (11)).

It will be noted that the placing of KHUFU and MENKAURA mirrors the subsidiary alignments. However, the east/west axis divides the figure into two seperate parts - to the south the radius of the circle is 1118 ($\sqrt{5}/2$ X 1000) cubits (Petrie's data giving 559 cubits east/west from the centre of KHAFRA to the west side of MENKAURA); while to the north the radius is the septenary 1120 cubits (Petrie's data giving 895.6 cubits north/south from the centre of KHAFRA to the north side of KHUFU suggesting an intended 896 cubits (4/5 X 1120), rather than the 894.4 cubits equal to $400\sqrt{5}$). This septenary figure produces three important whole-number ratios approximating $\sqrt{3}$: 97/56 71/41 26/15 , and shown together in the northwest quadrant of the figure. (The royal cubit is divided by a special mark into two parts of 13 and 15 digits so that it is possible for all these ratios to be produced by addition of segments). These ratios are reflected in the layout of various features at Giza; for example the ratio 71/41 (213/123) in the development of the initial plan. The 'mean' diameter of figure 7 is 2238 cubits - the same as that of the circle enclosing Legon's plan.

* * *

The chief results of the preceding analyses may be summarised as follows : the proportions of KHUFU, exhibiting the essential relations 220 + 280 = 500 and 220 + 213 = 433 (and reflecting the septenary dimensions of figure 7), are expanded by a factor of 5 to give the proportions of the initial plan for the three pyramids. The proportions of the final plan appear to incorporate the 'π' value 1273 cubits for the 'height' of the model pyramid, while the proportions of both KHUFU and KHAFRA, and the 'expanded' scheme for MENKAURA of figure 5, exhibit ϕ ratios expressed as whole numbers. The proportions of these plans ultimately reduce to a series of prime numbers, notably the set 1, 7, 19, and 37. The astronomical alignments exhibited in the design of the KHUFU starshafts (for example the altitude 39.5° for the culmination of Sirius) are also reflected in the layout (with Sirius rising at an azimuth 26.5°).

In view of the many mystical ideas attached to the pyramids scholars are understandably cautious when presented with complex geometric schemes like those cited in this paper, especially when the results appear to contradict the consensus view that the ancient Egyptians, 'although renowned in the ancient world for their cleverness', were 'not particularly given to abstract thought', and that the proportions found in architectural design 'can all be explained in terms of relatively simple practical procedures' (12). However, such pronouncements are primarily based upon examination of an extremely meagre collection of scribal exercise books post-dating the collapse of the Old Kingdom by many hundreds of years (13), and it is therefore preferable to draw conclusions from study of actual monuments.

But while the analyses described here may provide the Historian of mathematics with fresh evidence for discussion, clearly the architect must have had a higher purpose in the setting out of these geometrical plans than the mere demonstration of his mathematical skill. Equally, any future discussion of the cultic significance of Old Kingdom pyramids must take account of this new material if it is to have any real value. While the present writer has argued that Giza geometry is symbolic, representing what might be termed a 'protoPythagorean' system of design and the ability for metaphysical abstraction that this implies (14), only the stellar-correlation theory developed in the pages of this journal by Bauval has so far offered the possibility of a direct means of interpreting the geometry in terms of the cult, and it will be interesting to see what alternative hypotheses will be put forward to explain these new geometrical facts.

1. Bauval, R.G. 1989. 'A Master Plan for the Three Pyramids of Giza based on the Configuration of the Three Stars of the Belt of Orion'. DE.13.

2. Cook, R.J. 1994. 'The Stellar Geometry of the Great Pyramid'. DE.29.

3. Legon, J.A.R. 1988. 'A Ground Plan at Giza'. DE.10.

4. Petrie, W.M.F. 1883. The Pyramids and Temples of Gizeh. London.

5. Elsewhere he demonstrates convincingly the use of the $\sqrt{2}$ proportion (expressed as a diagonal of 99 units within a square of side 70 units and noticed by Petrie as determining the King's Chamber level within the Great Pyramid) in the spacing between the west sides of KHAFRA and MENKAURE, the layout of the Sphinx, the north/south division of the Giza layout plan by the south side of KHAFRA, and the same division at 1/10 scale to determine the height of the bend in the bent pyramid of DASHUR. With the demonstrated precision of all these latter results Legon effectively contradicts his own claim that an adjustment of one cubit was made to the initial plan for KHUFU and KHAFRA in order to improve the 'accuracy' of the $\sqrt{2}$ dimension in the overall site plan. If the 'root rectangle' theory thus falls by the wayside then we are left with no explanation whatsoever for the curious configuration of the Giza group.

The fact that the three pyramids were built along the strike of the bedrock, from northeast to southwest, does not explain the curious offset of the third pyramid.

6. The north/south dimension 676 cubits is produced by a construction within the layout square of side 1000 cubits from the centre of KHUFU : $1000 - 220 = 780$. $780/2 \times \sqrt{3} = 676$, employing the close approximation for $\sqrt{3}$ of 26/15 referred to later in this paper. (This is reiterated in the rectangle measuring 26 X 15 cubits formed between the southeast corner of the King's Chamber and the centre of the pyramid).

7. See Cook, R.J. 1992. The Pyramids of Giza. Glastonbury. Also see Legon, J.A.R. 1989 'The Geometry of the Great Pyramid'. Göttinger Miszellen.108. for an exceptionally precise description of the adjustments made to floorline lengths and passage angles in the final plan.

8. Legon, J.A.R. 1979. 'The Plan of the Giza Pyramids'. Staten Island Archaeological Reports. Volume 10. No. 1.

9. The use of the ϕ proportion to lay out ancient monuments cannot be demonstrated before the Minoan period (see Preziosi, D.A. 1968. 'Harmonic design in Minoan Architecture'. Fibonacci Quarterly.Vol.6). Attempts have been made to analyse ancient Egyptian architecture in a similar way (see Badawy, A. 1965. Ancient Egyptian Architectural Design. UCLA) but no consistent design rule has been isolated by such studies, while most of the material examined has been of New Kingdom date. On the other hand it is difficult to believe that the Giza architect would not have been familiar with the properties of simple additive series : the very dimensions of KHUFU (base 440, apothegm 356) exhibit the Fibonacci proportions 55:89. If the architect had wanted to design this pyramid geometrically he would most likely have used the ϕ construction made on the diagonal of the double square (leading directly to the construction of the pentagon and the form with which the ancient Egyptians represented stars) and which, according to the present writer, is implicit in the design of the starshafts (see note 2). It is noteworthy that both the general line of the passages within KHUFU and the layout alignments to the sun and Sirius are laid out on the diagonal of the double square, while the 345 meridian triangle of KHAFRA is determined by a geometrical construction based upon it.

The intervals 690 and 840 cubits seperating the northern sides of the three pyramids are simultaneously defined in other ways.

10. Legon, J.A.R. 1989. 'The Giza Ground Plan and Sphinx'. DE.14.

11. See Bauval, R. and A. Gilbert. 1994. The Orion Mystery. London. page 219.

12. Robins, G. and C.C.D. Shute. 1985. 'Mathematical Bases of Ancient Egyptian Architecture and Graphic Art'. HISTORIA MATHEMATICA. 12. page 119.

13. This is certainly not to beliittle the importance of these fragments in giving us some idea of the computational methods of the ancient Egyptians. This material is however almost entirely devoted to the needs of bureaucratic calculation and provides very few clues regarding what might be termed the 'religious mathematics' of Giza.

14. In a recent paper discussing how ancient Egyptians designed their buildings (Kemp, B. and P. Rose. 1993. 'Proportionality in Mind and Space in Ancient Egypt'. Cambridge Archaeological Journal.1(1).) the authors state (page 127) : 'Architects drew their inspiration from seeing what was around them but remained indifferent at an intellectual level to the workings of their own creative processes'...'It is this internal blindness that marks the Egyptians off so sharply from the Classical Greeks, something that, oddly, the Greeks themselves seem not to have perceived though they themselves claimed that Egypt was the original home of geometry'. We might perhaps gain useful insights into Egyptian methods if we occasionally took the Greeks at face value - they are unanimous in asserting that learning in the Egyptian temple was reserved for initiates. Plato studied there and, above the door of the first academy, had written the words 'Let none ignorant of geometry enter here'.

R. J. Cook Glastonbury. June 29 1994