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INHALT

THE GEOMETRY OF THE GREAT PYRAMID

John A.R. Legon

In a recent article describing the geometry of the Great Pyramid of Giza, Jorge Trench[1] has presented a model which, it is claimed, could have been used by the ancient Egyptian builders to determine the configuration of the passage-system. Although a high degree of accuracy is asserted, the validity of Trench's data may in some respects be disputed; while the complexity of the geometry may also render his model unlikely to have been envisaged by Khufu's architects of the Fourth Dynasty. The present paper puts forward an alternative development which is considerably more accurate, and also shows how many of the dimensions in the Great Pyramid were set to meaningful whole numbers of royal cubits.

To establish his geometrical model, Trench assumes that the sloping passages were all intended to have the same angle of slope, when the measured angles vary over a range of about half a degree, or from 26° 2' to 26° 34'. Within this range, many definitions of the slope are possible, and it is evident that each passage must be considered as a separate entity if any degree of accuracy is to be demonstrated. Trench also assumes that the angle of the Descending Passage given by J. and M. Edgar,[2] and referred to by Maragioglio and Rinaldi,[3] resulted from an actual measurement of the passage - when in fact this angle of 26° 18' is a hypothetical value derived from the 'pyramid-inch' model, which was based on the mean of the different passage-angles obtained by Piazzi Smyth.[4]

For the dimensions of the various passages, Trench refers to the measures of Maragioglio and Rinaldi,[5] which are mostly metric conversions of the careful measurements made by Petrie in inches.[6] Trench gives these measurements only after conversion to cubits, using a length of cubit that has evidently been allowed to vary from 0.5235 to 0.524 metres. Very few of the theoretical values correspond to whole numbers of cubits, and the lengths given for the Ascending Passage and Grand Gallery diverge from the measured dimensions by about 1.3 and 1.6 cubits respectively. As shown below, however, these lengths actually correspond to significant whole numbers of cubits to within 0.05 cubit.

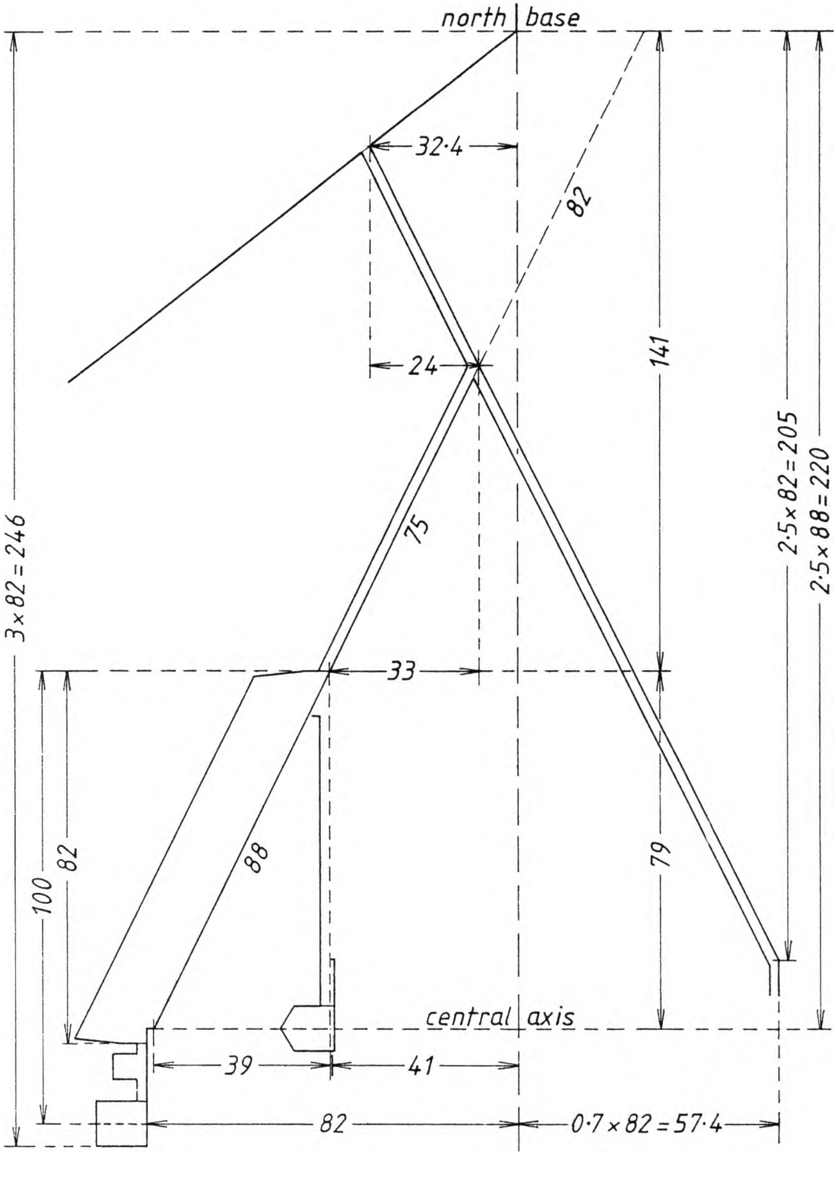

Fig.1 Dimensions of the Great Pyramid in Royal Cubits.

The Geometrical Design

The following description of the Great Pyramid's internal design extends that recently published by the writer,[7] including further details and a geometrical figure in which the main elements of the structure are exactly defined. Reference will be made to Petrie's measurements as originally stated by him in inches, and converted to royal cubits of fixed length 20.620 inches or 0.52375 metres, as also determined by Petrie.[8]

For the external form of the Great Pyramid, it is generally accepted that the base was set to 440 cubits, and that the chosen profile was 14 rise on 11 base with a seked of 5 1/2. The height was therefore just 280 cubits, for a casing-angle of 51° 50' 34" as compared to the measured angle of 51° 52'.[9] Petrie's finding for the mean side of base was confirmed to within 0.6 inch in the later survey by Cole,[10] his result being:[11]

Mean Base of Great Pyramid, 9068.8 ins = 439.81 cubits

Although reasons for the divergence from 440 cubits have been put forward,[12] the difference amounts to less than 0.05%.

Within the Great Pyramid (fig.1) a primary division is formed by the central axis, which is exactly marked by the apex of the Queen's Chamber and by the vertical face of the Great Step at the head of the Grand Gallery, as shown by Petrie's measures:

	Inches	Cubits
Face of Great Step, S. from N. base	4534.5	219.91
Queen's Chamber, Apex " " " "	4533.8	219.87
Mean semi-base of Great Pyramid	4534.4	219.90

A second major division is marked by the level of the King's Chamber; and we can confirm Petrie's finding that this chamber was placed at the height in the Great Pyramid at which the area of the horizontal section is equal to one-half the area of the base. Additionally, however, this level is found to correspond to a whole number of cubits - a number that also determined the choice of several other dimensions in the Great Pyramid. Geometrically, the upper height of the Pyramid above the floor of the King's Chamber is equal to the semi-diagonal of a square, the side of which is given by the total height of 280 cubits:

From King's Chamber to Apex = $280 \div \sqrt{2}$ = 197.99 cubits

Hence King's Chamber Level = 280 - 197.99 = 82.01 cubits

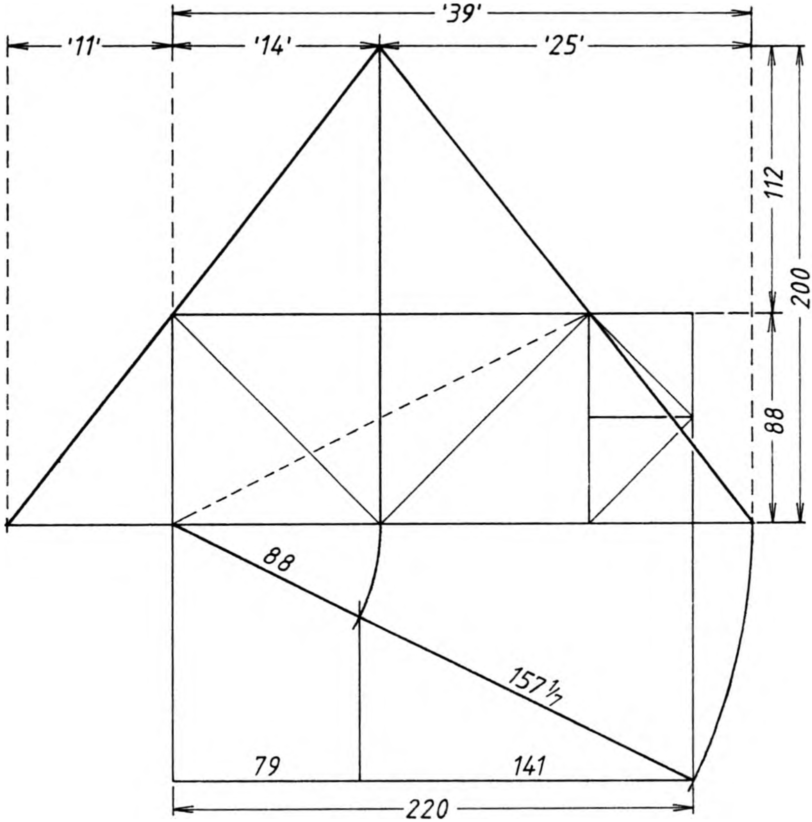

Fig.2 Geometrical Development of Dimensions in Great Pyramid.

Level of King's Chamber over Base, Petrie:

1691.4 to 1693.7 ins = 82.03 to 82.14 cubits

With reference to fig. 1, the importance of this level of just 82 cubits is shown by its use as a modulus in other dimensions:

	Inches	Cubits	Modulus
Horizontal Length of Grand Gallery	1688.9	81.90	x 1 = 82
N. end Ascending Pass. to N. base[13]	1691.0	82.01	x 1 = 82
S. Wall King's Chamber to N. base[14]	5071.1	245.93	x 3 = 246
S. end Descending Passage to N. base	4228	205.04	x 2.5 = 205
Pavement Level of Queen's Chamber[15]	844.2	40.94	x 0.5 = 41

These measurements show an average error of only 0.06 cubit.

Divisions of the Semi-Base and Lengths of the Passages

Between the north base of the Great Pyramid and the central plane, significant divisions occur at the north wall of the Grand Gallery, at the point of junction with the Ascending and Queen's Chamber Passages, and at the lower end of the Ascending Passage at its junction with the Descending Passage. These positions correspond to an exact division of dimensions in the ratio 14:(11 + 14) or 14:25, this ratio being simply derived from the pyramid-profile of 14 rise on 11 base. The semi-base of the Great Pyramid of 220 cubits is divided at the north wall of the Grand Gallery in the ratio 14:25 as follows:

$220 \times 14/(14 + 25) = 78.974$, $220 \times 25/(14 + 25) = 141.025...$

The two parts are therefore nearly whole numbers of cubits, the larger being exactly as shown by Petrie's placing of the Gallery:

N. wall of Grand Gallery from N. Base, 2907.3 ins = 140.99 cubits

At the junction between the Descending and Ascending Passages, the distance of 82 x 2.5 or 205 cubits horizontally from the north base of the Great Pyramid to the foot of the Descending Passage, is also divided in the ratio 14:25

$205 \times 25/(14 + 25) = 131.410$, $205 \times 14/(14 + 25) = 73.589...$

N. end Ascending Passage from N.Base, 1517.8 ins = 73.61 cubits

The error in this position is therefore less than 0.03 cubit.

The ratio of 14:25 can be developed together with the actual dimensions in the Great Pyramid as shown in figure 2, starting with a pyramid of the same 14:11 proportion but with a height of just 200 cubits. By forming squares within the pyramid-section, the base is divided into parts of 11, 14, 14 and 11, so that the origin of the ratio 14:(14 + 11) is clearly indicated. Figure 2 also gives the exact sloping length of the Grand Gallery, which from the junction with the Ascending Passage up to the Great Step is just 88 cubits, or two-fifths of the semi-base of 220 cubits from the north base to the Great Step. The length of 88 cubits is defined by the sides of the squares, which divide the height of 200 cubits in the ratio 11:14 as follows:

200 x 14/(11 + 14) = 112, 200 x 11/(11 + 14) = 88

Sloping length of Grand Gallery, 1815.5 ins = 88.05 cubits

The semi-base of the pyramid-section, which is 200 x 11/14 equals 157 1/7 cubits or 1100 palms, corresponds to the sloping distance from the north wall of the Grand Gallery to the vertical plane of the north base. The Ascending Passage represents just 75 cubits in this length, the remaining part being (157 1/7 - 75) or about 82 cubits, as previously noted. From Petrie's survey we have:

Sloping length of Ascending Passage, 1546.8 ins = 75.01 cubits

We can now construct the profile of the Ascending Passage and Gallery by applying the total sloping length of (88 + 157 1/7) or 88(1 + 25/14) equals 88 x 39/14 cubits, to the horizontal length of 88 x 5/2 equals 220 cubits. This profile reduces to 39 slope on 35 base, which is close to the mean of Petrie's data:

Angle of Slope, for 39 slope on 35 base = 26° 10' 37"

Mean Angle, Ascending Passage and Gallery = 26° 9' 35"

Petrie's measurements, however, suggest that the final angles of the Ascending Passage and Gallery were significantly adjusted with respect to their mean value, such that suitable whole numbers of cubits were obtained in the vertical lengths of both passages. From Petrie's survey we have:

	Inches	Cubits	Design
Vertical length of Grand Gallery[16]	803.8	38.98	39
Vertical length of Ascending Passage	679.7	32.96	33

The vertical length of the Gallery floor is therefore 39 cubits, with possible allusion to the (14 + 25) or 39 parts into which the semi-base of the Pyramid is divided. The resulting profile of 88 slope in 39 rise gives a theoretical angle of 26° 18' 25". The vertical length of the Ascending Passage is clearly 33 cubits, or 3/8 of the Gallery floor-length; and since the sloping length of this passage is 75 cubits, the passage-profile is 25 slope in 11 rise, with a theoretical angle of 26° 6' 14". Comparing these angles with the measures of Petrie, and also Smyth,[17] we have:

	Petrie	Smyth	Design
Ascending Passage	26° 2' 30"	26° 6' 0"	26° 6' 14"
Grand Gallery	26° 16' 40"	26° 17' 37"	26° 18' 25"

Hence the passage-sections of the Ascending Passage and Gallery are 75 cubits slope in 33 cubits rise, and 88 cubits slope in 39 cubits rise, respectively.

The Descending Passage

Reference to the series of offsets taken by Petrie to the floor of the Descending Passage,[18] from a mean axis of angle 26° 31' 23", shows that the slope of this passage is divided into two parts at the junction with the Ascending Passage. The slope of the lower part corresponds exactly to the simple profile of one rise on two base,[19] the two parts being:

Upper Slope of Descending Passage	=	26° 27' 41"
Lower Slope " " "	=	26° 33' 6"
Angle for Slope, for 1 rise on 2 base	=	26° 33' 54"

To explain the lesser angle of the upper part, it appears that an adjustment has been made to make the vertical length a whole number of cubits. Petrie's measure was 495.3 inches equals 24.02 cubits, which is just 1/3 of the vertical rise of (33 + 39) or 72 cubits through the Ascending Passage and Gallery. Petrie placed the junction with the Ascending Passage at a level over the base of 172.9 inches,[20] which is 8.39 or nearly 8.4 equals 7/10 x 12 cubits. Multiples of 12 cubits are thus evident in these vertical dimensions - the level of the entrance to the Great Pyramid being defined as 12(2 + 7/10) equals 32.40 cubits, or close to Petrie's placing at the level of 668.2 inches or 32.405 cubits. The foot of the Descending Passage is placed 1181 inches or 57.27 cubits below the base, this being about 7/10 x 82 equals 57.4 cubits.

Conclusions

While this description of the Great Pyramid's passage system does not represent a complete analysis, enough material has been put forward to show the great precision with which the dimensions of the passages were set - both in terms of the Egyptian cubit and with reference to a logical geometrical design. Some writers have stressed the use of the seked, or relation between the vertical and horizontal lengths of a slope, as if the ancient Egyptians were unable to determine the sloping length of a passage by direct measurement. Although the use of the Pythagorean theorem is still a matter of debate, any problems involving the hypotenuse of a right-angled triangle could have been solved by measurement in a plan drawn to a suitable scale. Whatever methods were used, the skill of the architect in selecting the sloping, horizontal, and vertical lengths of the passages, was clearly quite considerable.

J.A.R. Legon

NOTES

1 J.A. Trench, Göttinger Miszellen 102 (1988), 85-94.
2 J. & M. Edgar, The Great Pyramid Passages (Glasgow, 1924) II, 12.
3 V. Maragioglio and C.A. Rinaldi, L'Architettura delle Pirimidi Memphite, IV (Rapallo, 1965), 10.
4 C.P. Smyth, Life and Work at the Great Pyramid (Edinburgh, 1867).
5 Maragioglio and Rinaldi, op. cit., 5.
6 W.M.F. Petrie, The Pyramids and Temples of Gizeh (London, 1883).
7 J.A.R. Legon, Discussions in Egyptology 12 (1988), 41-48.
8 Petrie, op.cit., 179.
9 Ibid., 184.
10 J.H. Cole, The Determination of the Exact Size and Orientation of the Great Pyramid of Giza, Survey of Egypt paper 39 (1925).
11 For full survey data see Petrie, op. cit. 37-95 (1st ed. only).
12 See Petrie, op. cit., 220.
13 From horizontal of 1517.8 inches at mean angle of 26° 9' 35".
14 This is 537.0 + 4534.1 inches, from Petrie, op. cit., 83, 95.
15 Reconstructed by Petrie, op.cit. 187: the pavement is missing.
16 From the sloping length at Petrie's mean angle of 26° 16' 40".
17 Petrie, op. cit. 65, 71; Smyth, op. cit. II, 151, 161.
18 Petrie, op. cit. 58; Smyth, op. cit. II, 148, gives 26° 27'.
19 Petrie, op. cit. 191; see also J-Ph. Lauer, Le mystère des pyramides (Paris, 1974), 297.
20 Trench, op.cit., refers to the Edgars' hypothetical measure.