

DISCUSSIONS  
IN  
EGYPTOLOGY

10

1988

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## A GROUND PLAN AT GIZA

John A.R. Legon

Now that a detailed topographical study of the Giza Plateau is in progress [1], it is interesting to consider the results of the excavations and survey carried out by Flinders Petrie in 1880-2, when the exact dimensions and relative positions of the Pyramids of Khufu, Khaefre and Menkaure, were fixed by triangulation [2]. With reference to Petrie's survey-data, this paper reviews the evidence put forward by the writer in 1979, showing that the size and relative positions of the three pyramids were determined by a single unifying ground plan [3].

The existence of a dimensional scheme underlying the placing of the Giza Pyramids is indicated in the first instance by the regular arrangement of these pyramids on the Giza Plateau, by which the sides of the bases, and the distances that separate them, form consecutive axial distances from north to south and from east to west. The three pyramids are accurately aligned with respect to the four cardinal points, and the bases are displaced from one another in a formation that meets the requirements of a coherent dimensional relationship. Difficulties with the site chosen for each pyramid also suggest that there must have been some constraint in addition to the usual factors such as ease of construction or architectural setting.

### The Survey Data

Using some of the best equipment available in his day, Petrie claimed to have fixed the main stations of his triangulation to within 3 millimetres [4]; and his result for the mean side of the Great Pyramid differed from the value obtained in the later survey by J.H. Cole [5], by only 1.5 centimetres. The dimensions of the pyramid-bases are given in Table I as stated by Petrie in inches, together with the average variations in the lengths of sides, and the orientations of the three pyramids with respect to geographic north. The distances separating the centres of the pyramids, as computed by Petrie [6] along axes parallel to the mean azimuth of the Second and Great Pyramids of  $-4^{\circ} 52''$ , are

given in Table II.

To obtain the components of spacing between the bases of the three pyramids, the distances between the pyramid-centres can be combined with the sides of the bases to give the results listed in Table III. There will be small differences at the corners due to variations in the azimuths of the sides with respect to the axes of the plan; but as the Second and Great Pyramids have the same orientation within two minutes of arc - a remarkably small divergence - these differences average only about 5 cm. The Third Pyramid, however, differs in azimuth from the Second and Great Pyramids by about 1/3 degree in a clockwise direction, so that elements at the corners of about 25 cm are generated relative to the mean components of spacing. The exact variations are given by an analysis of Petrie's original coordinates of survey.

The various dimensions are now expressed in terms of the Royal Egyptian Cubit, with the value of 20.620 inches or 0.52375 metres as determined by Petrie from measurements in the Great Pyramid [7] and as stated by Edwards [8]. Almost all of the mean components of spacing correspond to the 'design' values listed in Table III within 0.1 cubit, the largest apparent difference being 0.23 cubit or 12 centimetres.

### **The Ground Plan**

To consider the possibility of positional relationships between the three pyramids, it is logical to assume that a dimensional scheme would have started from the base of the Great Pyramid, at the north-east corner of the plateau, from which point distances would have been measured along two axes southwards and westwards out to the base of the Third Pyramid. It is well-known that the sides of the Great Pyramid measure 440 cubits, although the actual mean side is 439.8 cubits for the cubit of 0.52375 metres. In the survey by Cole [9], only the longest (south) side has a length of exactly 440 cubits, the measured dimension being 230.454 metres; while the average variation in the lengths of the sides was about 6 cm. or 0.1 cubit. Petrie [10] thought that an adjustment may have been effected in order that the perimeter of the base should express the so-called 'pi-proportion' in relation to the height of 280 cubits, with greater accuracy than the value for pi of 22/7.

Turning now to the relative position of the Second Pyramid, we firstly find that the north side of this pyramid is placed by the survey-data 250.23 cubits southwards from the south side of the Great Pyramid. It is therefore suggested that the builders actually intended a north-south spacing between the two pyramids of just 250 cubits. Taking further distances along the north-south axis, the impression of a deliberate intention in the position of the Second Pyramid is strongly supported. The distance southwards from the north side of the Great Pyramid to the south side of the Second Pyramid is 1101.04 cubits, or only 0.1% greater than the round-figure distance of 1100 cubits. This is  $2 \frac{1}{2}$  times the side of the Great Pyramid of 440 cubits, so that the axial north-south distance between the south sides of the two Pyramids appears to have been laid out as simply  $\frac{3}{2} \times 440$  or 660 cubits.

At this point we can derive a value for the side of the Second Pyramid, from the formative dimensions of 440 and 250 cubits:

Initial Side for Second Pyramid =  $440 \times \frac{3}{2} - 250 = 410$  cubits  
 This is termed the initial side, because Petrie's survey gives an actual mean side of 8474.9 inches or exactly 411 cubits, with an average variation in the sides of only 1.5 inches or 4 cm. Thus although the above derivation accounts for a side of about 410 cubits in preference to one of say 400 cubits, an adjustment of one cubit has evidently been made in the final base of the Second Pyramid. Reasons for this adjustment will be given shortly.

Taking dimensions on the east-west axis, the west side of the Second Pyramid is placed by Petrie's data, 624.09 cubits westwards from the west side of the Great Pyramid. Again with an adjustment of one cubit, this dimension may clearly be ascribed an initial value of 625 or  $2 \frac{1}{2} \times 250$  cubits, making it analogous to the distance of  $2 \frac{1}{2} \times 440$  cubits along the north-south axis.

The size and relative placing of the Second and Great Pyramids are thus explained by an elementary scheme based on modules of 440 and 250 cubits, in which two and a half squares of 440 cubits are placed from north to south to define the base of the Great Pyramid and the position of Second Pyramid at its south side; while two and a half squares of 250 cubits are arranged from east to west to give the north-south spacing between the two pyramids of 250 cubits, and the east-west dimension of  $2 \frac{1}{2} \times 250$  equals

625 cubits (see figure 1). The base of the Second Pyramid is then defined as  $(660 - 250)$  or 410 cubits, and the east-west spacing from the Great Pyramid, as  $(625 - 410)$  or 215 cubits.

With reference to this initial scheme, however, the builders evidently subtracted one cubit from the dimension of 625 cubits, and added one cubit to the dimension of 1100 cubits, making the side of the Second Pyramid 411 cubits and its east-west spacing from the Great Pyramid,  $(624 - 411)$  equals 213 cubits (see Table III). These adjustments suggest that some additional factors may have influenced the final choice of dimensions; and indeed these factors are now found to derive from the inclusion of the Third Pyramid in the ground plan.

As shown by the survey-data in Table III, the Third Pyramid extends the scheme of the Second and Great Pyramids by 631 cubits towards the south and 353.5 cubits towards the west, and sets the overall dimensions of the ground plan (figure 1). On the east-west axis, the axial distance of 353.5 cubits from the west side of the Second Pyramid to the west side of the Third Pyramid is in effect already given in the plan, since it is exactly the diagonal in a square of side 250 cubits:

$$250 \times \sqrt{2} = 353.55\dots \text{ cubits}$$

This is also about one-quarter of the overall east-west dimension of the plan from the east side of the Great Pyramid to the west side of the Third Pyramid, which is approximately  $1000 \sqrt{2}$  equals 1414.2... cubits:

Overall East-West Dimension =  $440 + 624 + 353.5 = 1417.5$  cubits  
It will be noted that the adjustment of one cubit in the component part of 625 cubits improves the accuracy of this dimension, in its equivalence to the diagonal in a square of 1000 cubits.

On the north-south axis, the distance of 631 cubits between the south sides of the Second and Third Pyramids is equal to the side of the Second Pyramid plus the semi-side of the Great Pyramid, or  $(411 + 220)$  cubits. The overall dimension from the north side of the Great Pyramid to the south side of the Third Pyramid is therefore:

$$\text{Overall North-South Dimension} = 1101 + 631 = 1732 \text{ cubits}$$

This dimension perfectly expresses the number  $1000 \sqrt{3}$  equals 1732.0..., and complements the east-west dimension of  $1000 \sqrt{2}$

cubits. It is the diagonal in a rectangle measuring 1000 cubits by  $1000\sqrt{2}$  cubits, so that the overall dimensions along the two axes of the plan are readily constructed from a square of side 1000 cubits. The use of these dimensions implies that the builders could calculate the numerical values of square-roots, a fact which is already well-known [11]; while the adjustments of one cubit shown in the distances of 624, 631 and 1101 cubits, all served to improve the geometrical accuracy of the overall dimensions.

Petrie obtained a mean side for the Third Pyramid of 4153.6 inches equals 201.44 cubits, which suggests a nominal dimension of 200 cubits adjusted by 1.5 cubits. This resulted in a spacing from the Second Pyramid of  $(353.5 - 201.5)$  or 152 cubits equals 1064 palms, the number of palms being the same as the number of cubits in the further distance along the same axis from the west side of the Second Pyramid to the east side of the Great Pyramid, of  $(624 + 440)$  equals 1064 cubits. At the same time, significant round tens of cubits were produced in distances relative to the Second Pyramid, taking account of the fact that the Third Pyramid shows a clockwise rotation with respect to the axes of the plan of 19 minutes of arc, such that the corners are displaced from the mean positions of the sides by just 0.5 cubit.

Whilst, therefore, the mean north-south spacing between the Second and Third Pyramids is  $(631 - 201.5)$  or 429.5 cubits, the north-east corner of the Third Pyramid is in fact 430 cubits south from the south side of the Second Pyramid; and this is twice the initial east-west spacing between the Second and Great Pyramids of 215 cubits. The north-west corner of the Third Pyramid, on the other hand, is  $(429 + 411)$  or 840 cubits south from the north side of the Second Pyramid, a dimension equal to three times the height of the Great Pyramid of 280 cubits.

### Conclusions

This description has been confined to the major factors underlying the size and relative positions of the Giza Pyramids, sufficient to show the inherent simplicity and significance of the various relationships. A number of further factors might be mentioned, however, and it must be noted that other structures were included in the ground plan, and influenced the choice of dimensions.

The placing of the three pyramids in a single ground plan was obviously an ambitious project, and one which indicates that the architects and builders of the Fourth Dynasty had a much greater control over the design and construction of these royal edifices than has hitherto been recognised. They were apparently able to dictate, for example, the small dimensions of the Third Pyramid, despite the presumed desire of Menkaure to have a monument equal to those of his predecessors. Since the three large pyramids of Meydum and Dahshur appear all to have been built by Sneferu [12], it seems possible that at the outset, Khufu himself might have aspired to the construction of the three Pyramids of Giza in a single unifying ground plan.

J.A.R. Legon.

#### NOTES

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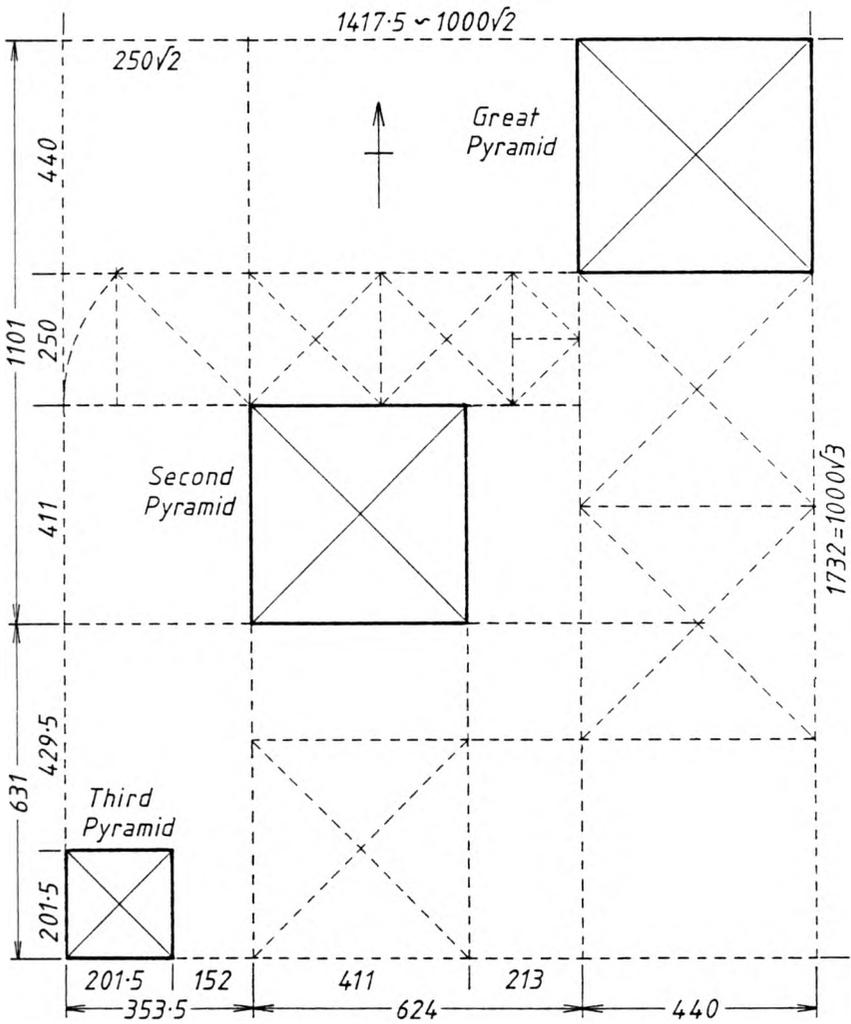


Figure 1. Dimensions of the Ground Plan in Cubits.

**TABLE I**

Mean Sides of the Giza Pyramids,  
as stated in Inches by Petrie and in Cubits of 20.620 inches:

	Inches	+/-	Cubits	Azimuth
Great Pyramid	9068.8	0.6	439.81	-3' 43"
Second "	8474.9	1.5	411.00	-5' 26"
Third "	4153.6	3.0	201.44	+14' 03"

(Cole:

Great Pyramid	9069.4	2.3	439.84	-3' 07")
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**TABLE II**

Axial Distances between the Centres of the Three Pyramids,  
as stated in Inches by Petrie:

From	N to S	E to W
Centre of 1st to Centre of 2nd Pyramid	13931.6	13165.8
Centre of 2nd to Centre of 3rd Pyramid	15170.4	9450.2
Centre of 1st to Centre of 3rd pyramid	29102.0	22616.0

**TABLE III**

Principal Components of Spacing between the Sides  
of the Three Pyramids, from the above data:

Axial Distance from North to South	Inches	Cubits	Design
N side 1st to S side 2nd Pyramid	22703.4	1101.04	1101
S " 2nd to S " 3rd "	13009.7	630.93	631
N " 1st to S " 3rd "	35713.2	1731.97	1732
S " 1st to N " 2nd "	5159.7	250.23	250
S " 2nd to N " 3rd "	8856.1	429.49	429.5
Axial Distance from East to West			
W side 1st to W side 2nd Pyramid	12868.8	624.09	624
W " 2nd to W " 3rd "	7289.5	353.52	353.5
E " 1st to W " 3rd "	29227.2	1417.42	1417.5
W " 1st to E " 2nd "	4393.9	213.09	213
W " 2nd to E " 3rd "	3135.9	152.08	152